

Decreasing k by unity also occurs to ensure that the last extreme which is to be stored is a similar extreme as the very first one. This is done because the frequency of each oscillation is determined from the distance of two successive similar extremes, as has been mentioned before. The selection process is completed either if all (n) elements of array A have been considered or if the values of m and j have increased such that array A is exhausted.

Figure 2 shows the effects that the MDA, MRA, and mean signal-level values have on the results of the final selection process. It is seen from Fig. 2a, which applies to a high mean signal-level, that a variation of the value for the MDA does not severely affect the selection process, but the value for the MRA, which dominates in this case causes significant differences. The opposite, i.e., the domination of the MDA criterion at low mean signal-levels is clearly illustrated by means of Fig. 2b.

Conclusion

A novel procedure has been developed to determine in an efficient, digital manner the amplitudes and frequencies of oscillatory components of arbitrary signals. Especially in those cases where varying amplitudes and frequencies are encountered, the method proves to be very useful.² The possibility to assign a noise level criterion (MDA-MRA) considerably enlarges the versatility of this procedure in comparison with other methods. Even for large amounts of data, computer times remain limited.

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Finite Volume Method for Blade-to-Blade Flows Using a Body-Fitted Mesh

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Introduction

THE time-dependent finite volume method provides a means of treating the problem of blade-to-blade flows in turbomachinery as an initial-value problem, since the equations for unsteady flows are always hyperbolic with respect to time in both subsonic and supersonic flow regions. On the other hand, the computation of steady flows by a time-dependent method differs from ordinary initial-value

problems in that the initial data and much of the transient solution have a negligible effect on the final or stationary solution.

The integral method first described by McDonald¹ and then modified by Denton² and Van Hove³ is more stable than the differential methods in Refs. 3 and 4 since all fluxes are conserved. Also, the integral method overcomes the difficulties due to the complex geometries encountered in practical turbomachinery problems. The method can readily be applied to shocked flows, where the shock capturing comes through an implicit artificial viscosity. The limit on the time step imposed by the stability requirement is the main disadvantage of time-marching methods.

The objective of this paper is to describe an efficient and accurate approach to the time-dependent finite-volume method applied to blade-to-blade flows in two dimensions. The approach developed consists of solving the Euler equations in conservation law form on a curvilinear body-fitted mesh.

The flow is assumed to be homenergetic, so that the energy equation can be omitted. However, homenergetic flow is only physical in the asymptotic limit of steady state. The time-marching computation will not represent a true unsteady phenomena, and this assumption is only used to reduce the computing time. This is made possible by the fact that a sound wave in a homenergetic flow travels with a lower speed than in an isentropic flow. Furthermore, this model requires one less variable.

Body-Fitted Curvilinear Meshes

In the application of the finite volume method, one generally assumes, for convenience, that the mesh is aligned with the flowfield, and this, to a large extent, will determine the accuracy of the solution. Denton's grid is the most widely used due to its simplicity. Generally, the grids used by previous authors are not very well aligned, and to increase the accuracy one must resort to the use of a finer grid, especially in the critical regions of the flowfield. These are regions where the properties vary rapidly such as the leading and trailing edges, and where the flow alignment with the grid is bad. Such grid refinement will result in an increase in computer time, since a sound wave cannot travel more than one mesh width in one time increment, which is the required condition for stability. This increase of computer time will be very pronounced in the case of a uniform time step, while in the case of a nonuniform time step, the increase will be directly proportional to the ratio between the number of nodes in the fine regions to the total number of nodes. Figure 1 shows a refined Denton grid.

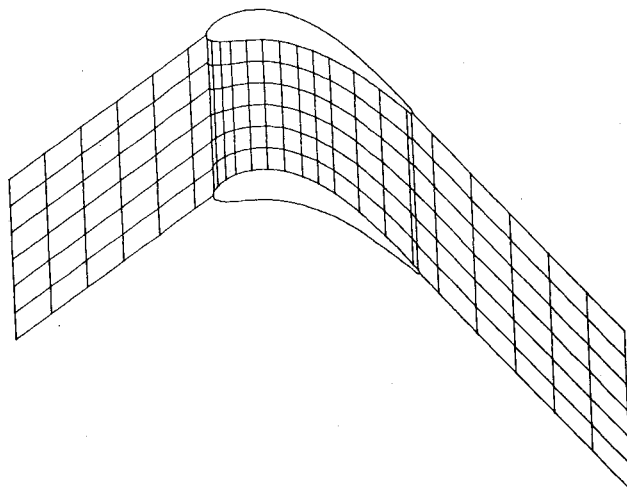


Fig. 1 Denton's grid (with concentration of nodes on the leading and trailing edges).

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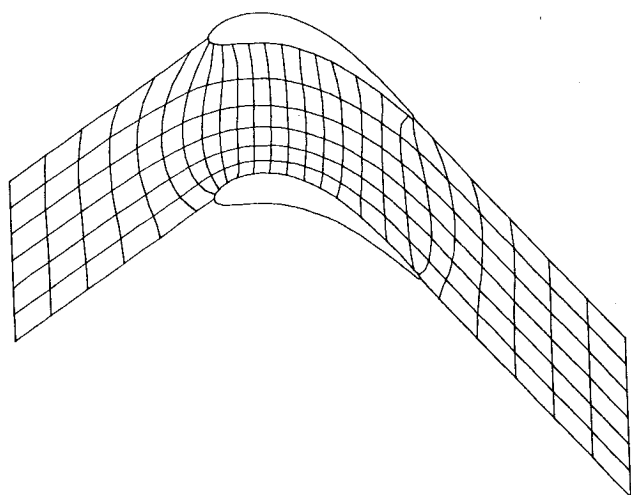


Fig. 2 Curvilinear body-fitted grid (with some concentration as in Fig. 1). Only every second grid line is shown.

In the present study it is proposed to use a curvilinear body-fitted mesh to overcome these drawbacks. By fitting the grid to the body shape and to the inlet and exit streams, one will obtain a better quasi-streamline in the computational domain and hence a better accuracy. These grids are generated by solving a system of nonlinear elliptic equations. The solution is obtained by a line relaxation scheme accelerated by a multigrid process, as described in Ref. 5. This procedure is very efficient and the computing time required is negligible compared to a subsequent time-marching scheme. Figure 2 shows such a curvilinear mesh with the same number of nodes as that shown in Fig. 1. The finer the grid near the streamline boundaries, the less error will be introduced to the computational domain. To get a finer grid near the streamline boundaries using Denton's grid, one would have to increase the number of the quasi-streamlines in the field, due to the equal spacing between the quasi-streamlines, and hence increase the computer time.

Numerical Scheme

The numerical scheme consists of applying the governing equations to an overlapped control volume in the pitchwise direction. Basically, the algorithm for a time step starts by a mass balance over the control volume faces to update the density over the entire computational domain. The new density, together with the old velocities, are used to update the pressure field. The periodicity condition is then applied on the pressure terms on the streamline boundaries outside the blade passage. The momentum fluxes in the pitch and stream directions are then updated by a momentum balance over the control volume faces, whereby the new pressure, old density, and old momentum fluxes are used. At the end of each time sweep, the periodicity is enforced by averaging the mass and momentum fluxes on the streamline boundaries outside the blade passage. On the solid boundaries, the slip condition is applied by aligning the velocity with the blade, and then the momentum fluxes are corrected accordingly.

Numerical stability is achieved by upwinding as suggested by Ref. 2. The momentum fluxes through the pitchwise faces of the control volumes are obtained from the upwind mesh, whereas the pressures are obtained from the downwind mesh. This process is particularly effective in the present method because of the good characteristics of the body-fitted grid.

Results

The present approach was applied to the computations of the blade-to-blade flows in turbomachinery applications. The VKI steam turbine tip section geometry⁶ was chosen as a test case, as it is regarded as the most severe test for a numerical

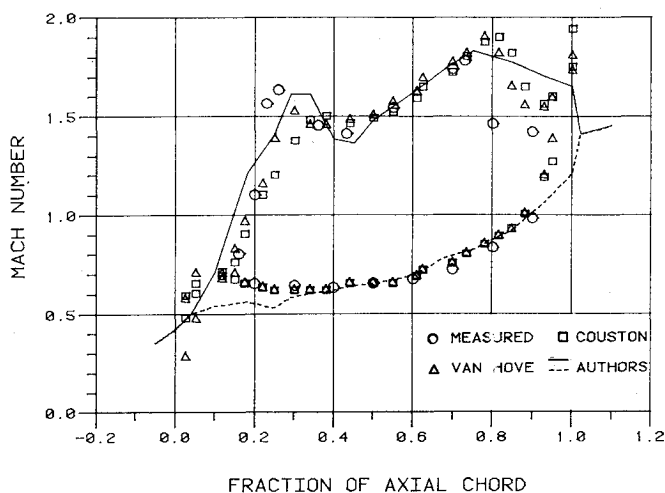


Fig. 3 Comparison of blade surface Mach number with experiments and numerical schemes.

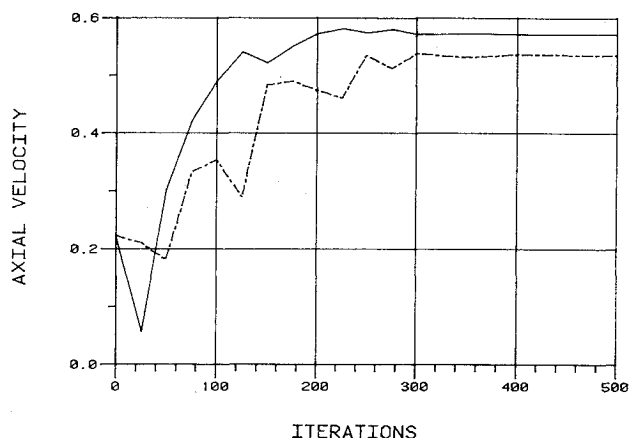


Fig. 4 Effect of axial pressure smoothing on the convergence of the time-marching scheme, shown on the axial velocity at the trailing edge.

scheme for transonic flow computations. Comparisons were made with experimental results as well as numerical results obtained by other schemes.

A body-fitted mesh of the above turbine was generated using 43×11 grid points (11 quasistreamlines and 43 pitchwise lines) similar to that shown in Fig. 2. The solution is presented in Fig. 3 as the blade surface Mach number distribution for an isentropic Mach number of 1.6. Also shown on this same figure are the experimental results of Ref. 6 and the numerical results of Couston⁷ and Van Hove.³ The present results yield a somewhat better prediction of the suction peak near the leading edge than the other two numerical schemes. It is noted that these used a finer grid, i.e., 61×15 and 50×20 , respectively. Since these are all essentially variations of the integral control volume of Ref. 1, the improvement can be attributed to the curvilinear mesh used in the present approach. This was confirmed by numerical experiments using the present finite-volume scheme together with body-fitted and nonbody-fitted grids (i.e., Denton's grid) as shown in Figs. 2 and 1, respectively. It was found that the solution using curvilinear grids converged four times faster. This is explained by the shape of the control volumes, which for the curvilinear grids have an aspect ratio close to 1:1 allowing a larger time step.

And another factor affecting the convergence was found to be the manner in which the periodicity was implemented; the periodicity condition on the pressure should be enforced after the pressure is updated rather than at the completion of a time

step. Convergence is also dependent on such numerical techniques as smoothing the pressure in the axial direction using a relation of the form

$$P_{\text{new},ij} = P_{ij} + \alpha(P_{i-1,j} + P_{i+1,j})$$

where P_{ij} represents the pressure at the i th axial node and j th pitch node. This is carried out for each node after each time step. The smoothing factor α depends on the particular grid and on the conditions. The selection of an "optimum" value is a trial-and-error process which can be quite costly. A good value was found to be about 0.1 for the present case.

Figure 4 shows the effect of smoothing on the number of iterations required to reach the final stationary solution. The reference property is taken as the nondimensional axial velocity component at the trailing edge on the suction side. Comparison shows that smoothing of the pressure reduced the calculation time by 1/8, taking into account the extra work required to smooth the pressure.

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External/Base Burning for Base Drag Reduction at Mach 3

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Introduction

EXPERIMENTS have shown that base burning can result in significant base drag reduction at supersonic speeds with good fuel efficiency.¹⁻⁶ Since efficiency decreases with increasing base pressure, the application of pure base burning seems to be limited to base drag reduction (i.e., to base pressures near or below the freestream pressure). It is known that net base thrust is possible using external burning.^{7,8}

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However, since base burning expands the wake gases and drastically reduces the wake momentum flux, external burning in combination with base burning offers the best promise for achieving base thrust. In fact, there is some experimental evidence that combined external and base burning may be more efficient for base drag reduction than base burning alone.⁵ On the other hand, it has been shown recently, using a simplified analysis for the case of net base thrust, that combined external and base burning is not attractive from a fuel usage point of view.⁹ Additional experiments are needed in order to evaluate the potential of combined external and base burning.

For several years the authors have been engaged in studies related to understanding and developing base and/or external burning.^{6-8,10,11} The purpose of the Note is to disclose recent results of tests with combined base and external burning using pure hydrogen as the fuel.

Test Facility

The blowdown-type test facility simulates the base flow for an axisymmetric projectile at Mach 3. A schematic of the test section is shown in Fig. 1. The hollow cylindrical model is supported in the ducting upstream of the nozzle throat, virtually eliminating support effects. Hydrogen and instrumentation leads are ducted into the model through the four support struts. The hydrogen is at ambient temperature. The stagnation temperature of the tunnel air flow drifts downward from about 10 to -20°C during a typical run. The stagnation pressure of the tunnel flow is maintained at about 7000-mm Hg by a pressure regulator.

Three base configurations were used for the tests reported herein. These are shown in Fig. 2. The top panel shows the base injection configuration used for pure base burning. This configuration is shown installed in the schematic of Fig. 1. It used a porous sintered-metal base plate for axial injection of a uniform stream of hydrogen. The other two panels show the configurations used for combined base and radial injection. One configuration uses a channel and the other, a step, as flameholders around the six equally spaced jet nozzles. The nozzles are constant diameter orifices drilled radially inward into the hollow centerbody. Both of these configurations are also fitted with sintered-metal base plates for simultaneous axial injection into the near wake.

The combustible mixture in the near wake was ignited by a consumable pyrotechnic igniter attached to the base. Combustion in the channel flame holder was initiated separately by a coating of pyrotechnic compound in the channel.

Results and Discussion

The test results are presented in Fig. 3, where the fuel specific impulse is plotted against the increase in base force due to burning (i.e., the change in base force due to injection

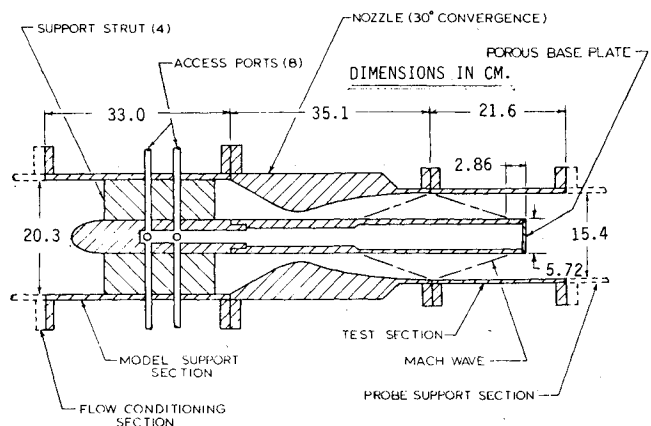


Fig. 1 Test section schematic.